Recursive and Trellis-Based Feedback Reduction for MIMO-OFDM with Rate-Limited Feedback

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***Abstract*— We investigate an adaptive MIMO-OFDM system with a feedback link that can only convey a finite number of bits. We consider three different transmitter configurations: i) beamforming applied per OFDM subcarrier, ii) precoded spatial multiplexing applied per subcarrier, and iii) precoded orthogonal space time block coding applied per subcarrier. Depending on the channel realization, the receiver selects the optimal beamforming vector or precoding matrix from a finite-size codebook on each subcarrier, and informs the transmitter through finite-rate feedback. Exploiting the fact that the channel responses across OFDM subcarriers are correlated, we propose two methods to reduce the amount of feedback. One is recursive feedback encoding that selects the optimal beamforming/precoding choices sequentially across the subcarriers, and adopts a smaller-size time-varying codebook per subcarrier depending on prior decisions. The other is trellis-based feedback encoding that selects the optimal decisions for all subcarriers at once along a trellis structure via the Viterbi algorithm. Our methods are applicable to different transmitter configurations in a unified fashion. Simulation results demonstrate that the trellis-based approach outperforms the recursive method as well as an existing interpolation-based alternative at high signal-to-noise-ratio, as the latter suffers from "diversity loss."**

***Index Terms*— Finite-rate feedback, multi-input multi-output (MIMO), OFDM, recursive vector quantization, trellis coded quantization.**

I. Introduction

MULTI-ANTENNA communications have attracted a tremendous amount of attention lately because of their promise of high transmission rate and much improved performance in fading channels. On the other hand, orthogonal frequency division multiplexing (OFDM) modulation has prevailed in recent broadband wireless systems, as it enables low complexity equalization for highly dispersive channels. The wedding of multi-antenna and OFDM leads to an appealing system design, termed multi-input multi-output (MIMO) OFDM, for high rate applications.

Adaptive transmissions can further improve system performance by matching transmission parameters to fading channels. Essential to adaptive transmissions is a feedback link from the receiver to the transmitter, although this is usually bandwidth limited and susceptible to error and delays. Adaptive MIMO-OFDM has been pursued based on delayed feedback in [8], and rate-limited feedback in [3], [4]. Specifically, transmit beamforming is deployed in [3] while precoded spatial multiplexing is used in [4] on each OFDM subcarrier, where the beamforming vector or the precoding matrix adapts to the channel based on a finite number of feedback bits. Exploiting the channel correlation across OFDM subcarriers, an interpolation-based approach is developed in [3], [4] to reduce the amount of feedback considerably.

As in [3], [4], we in this paper address feedback reduction for MIMO-OFDM. We consider three different transmitter configurations: i) beamforming applied per subcarrier, ii) precoded spatial multiplexing applied per subcarrier, and iii) precoded orthogonal space time block coding (OSTBC) applied per subcarrier. Linking feedback reduction to a compression type of problem for correlated sources, we propose two methods for feedback reduction relying on tools from the vector quantization literature [7]. One is recursive feedback encoding that selects the optimal beamforming vectors or precoding matrices sequentially across the subcarriers, and adopts a smaller-size time-varying codebook per subcarrier depending on prior decisions. The other is trellis-based feedback encoding that selects the optimal beamforming/precoding decisions for all subcarriers at once along a trellis structure via the Viterbi algorithm. Trellis-based feedback encoding outperforms recursive feedback encoding at the expense of complexity increase at the receiver.

At the outset, let us lay out distinctions of our methods relative to existing alternatives in [3], [4].

- In the high SNR region, the bit-error-rate (BER) curves of the interpolation-based approach level off, indicating "diversity loss", as the diversity order is exactly the slope of the BER-SNR curves in the log-log scale plot. This is not the case for our proposed trellis based approach. Our numerical results demonstrate that the trellis-based approach is inferior to interpolation-based alternatives at low SNR, but outperforms the latter considerably at the medium to high SNR range.
- Beamforming and precoded spatial multiplexing have been treated separately in [3] and [4], where the matrix interpolation requires special designs different from the vector interpolation. We have a unified treatment for beamforming, precoded spatial multiplexing, and precoded OSTBC.
- The beamforming vectors and precoding matrices across all subcarriers are drawn from a prescribed codebook in our proposed methods. On the contrary, the interpolated beamforming vectors and precoding matrices fall outside the given codebook in general [3], [4].

Notation: Bold upper and lower letters denote matrices

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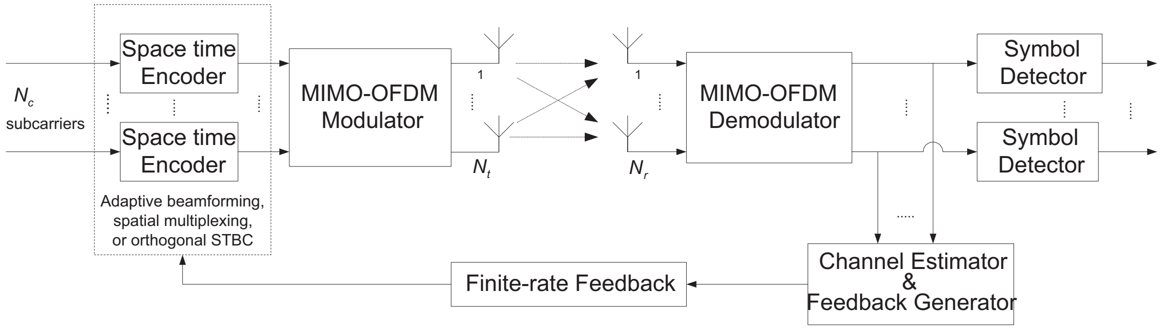

Fig. 1. The MIMO-OFDM system with adaptive space time encoder per subcarrier based on finite-rate feedback.

and column vectors, respectively; $(\cdot)^T$, $(\cdot)^*$, and $(\cdot)^{\mathcal{H}}$ denote transpose, conjugate, and Hermitian transpose, respectively; $\|\cdot\|_F$ is the Frobenius norm of a matrix. $\mathbf{I}_N$ is the $N \times N$ identity matrix; $[\mathbf{A}]_{i,j}$ stands for the $(i,j)^{\text{th}}$ entry of a matrix $\mathbf{A}$.

II. System Model

As depicted in Fig. 1, we consider a multi-antenna wireless communication system with N_t transmit- and N_r receive-antennas, where OFDM utilizing N_c subcarriers is employed per antenna transmission. We assume that data transmission occurs in a burst by burst fashion, and that the channels remain constant during each burst. Within one data burst, the fading channel between the μth transmit-antenna and the νth receive-antenna can be described by a linear filter with $L+1$ taps $\{h_{\nu\mu}(0), \ldots, h_{\nu\mu}(L)\}$ in discrete-time baseband representation, where L is the channel order. With p denoting the OFDM subcarrier index, the frequency response between the μth transmit- and the νth receive-antennas on the pth subcarrier is

$$H_{\nu\mu}[p] = \sum_{l=0}^{L} h_{\nu\mu}(l) e^{-j2\pi pl/N_c}, \quad p = 0, \ldots, N_c - 1. \quad (1)$$

At the pth subcarrier of the nth OFDM symbol, we collect the transmitted symbols across N_t transmit-antennas in an $N_t \times 1$ vector $\mathbf{x}[n;p]$, and the received symbols across N_r receive-antennas in an $N_r \times 1$ vector $\mathbf{y}[n;p]$. The channel input-output relationship on the pth subcarrier is then

$$\mathbf{y}[n;p] = \mathbf{H}[p]\mathbf{x}[n;p] + \mathbf{v}[n;p], \quad (2)$$

where $\mathbf{v}[n;p]$ is additive white Gaussian noise (AWGN) with each entry having variance N_0, and $\mathbf{H}[p]$ is the $N_r \times N_t$ channel matrix with the (ν,μ)th entry being $H_{\nu\mu}[p]$.

A. Three Transmitter Configurations

We consider three different transmitter configurations: i) beamforming applied per subcarrier, ii) precoded spatial multiplexing applied per subcarrier, and iii) precoded OSTBC applied per subcarrier.

We start with precoded spatial multiplexing, as used in [4]. At the pth subcarrier, collect N_s information symbols in an $N_s \times 1$ vector $\mathbf{s}[n;p]$. The symbol vector $\mathbf{s}[n;p]$ will be precoded by a matrix $\mathbf{T}[p]$ of size $N_t \times N_s$ to form the transmitted block $\mathbf{x}[n;p] = \mathbf{T}[p]\mathbf{s}[n;p]$. As a result, the channel input-output relationship in (2) becomes

$$\mathbf{y}[n;p] = \mathbf{H}[p]\mathbf{T}[p]\mathbf{s}[n;p] + \mathbf{v}[n;p]. \quad (3)$$

Various receiver structures can be adopted to demodulate $\mathbf{s}[n;p]$ from $\mathbf{y}[n;p]$. We here only consider a linear minimum-mean-square-error (MMSE) receiver. For the kth symbol, the signal to interference-plus-noise ratio (SINR) after the MMSE equalization is (see [10] for a detailed presentation):

$$\gamma_k^{\text{mmse}}[p] = \frac{E_s/N_0}{[\mathbf{T}^{\mathcal{H}}[p]\mathbf{H}^{\mathcal{H}}[p]\mathbf{H}[p]\mathbf{T}[p] + (N_0/E_s)\mathbf{I}_{N_s}]^{-1}_{k,k}} - 1, \quad k = 1, \ldots, N_s. \quad (4)$$

Let $\phi(\gamma)$ denote the relationship between the bit error rate (BER) and the signal-to-noise-ratio (SNR) γ in an AWGN channel; the closed-form expression for $\phi(\gamma)$ can be found in e.g., [2]. The average BER on the pth subcarrier is

$$\text{BER}[p] = \frac{1}{N_s}\sum_{k=1}^{N_s} \phi(\gamma_k^{\text{mmse}}[p]). \quad (5)$$

Averaging over N_c subcarriers, the average BER for the MIMO-OFDM system is

$$\overline{\text{BER}} = \frac{1}{N_c}\sum_{p=0}^{N_c-1} \text{BER}[p]. \quad (6)$$

We next consider transmit beamforming on each OFDM subcarrier, as used in [3]. This is actually a special case of precoded spatial multiplexing from setting $N_s = 1$. With transmit beamforming, the matrix $\mathbf{T}[p]$ reduces to a vector, and no matrix inversion is involved in (4).

We now consider precoded OSTBC on each OFDM subcarrier, as used in [8]. For brevity, we just illustrate the results with the Alamouti code [1]. On each subcarrier, a 2×2 Alamouti code matrix is constructed, which is then precoded by a $N_t \times 2$ matrix $\mathbf{T}[p]$, to obtain the transmitted blocks $\mathbf{x}[2n;p]$ and $\mathbf{x}[2n+1;p]$ for two consecutive OFDM symbols. Specifically, with two symbols $s[2n;p]$ and $s[2n+1;p]$, the transmitter constructs

$$\left[\mathbf{x}[2n;p], \mathbf{x}[2n+1;p]\right] = \mathbf{T}[p]\begin{bmatrix} s[2n;p] & -s^*[2n+1;p] \\ s[2n+1;p] & s^*[2n;p] \end{bmatrix}. \quad (7)$$

The optimal receiver applies linear processing on $\mathbf{y}[2n;p]$ and $\mathbf{y}[2n+1;p]$ to separate the detection of $s[2n;p]$ and $s[2n+1;p]$, as detailed in [1]. The SNR for detecting each information symbol is [8]:

$$\gamma^{\text{ostbc}}[p] = \frac{E_s}{N_0}\|\mathbf{H}[p]\mathbf{T}[p]\|_F^2. \quad (8)$$

The BER per subcarrier is then $\text{BER}[p] = \phi(\gamma^{\text{ostbc}}[p])$. Based on (6), we obtain the BER for MIMO-OFDM based on precoded OSTBC.

B. Per-Subcarrier Feedback

We assume an error-free delay-free feedback link from the receiver to the transmitter, which can only convey a finite number of bits per feedback interval. With limited feedback, we will draw $\mathbf{T}[p]$ from a finite-size codebook with N elements $\mathcal{T} := \{\mathbf{T}_1, \ldots, \mathbf{T}_N\}$.

MIMO-OFDM yields N_c parallel flat-fading subchannels. If one *ignores* the channel dependence across subcarriers, the optimal precoding matrices will be selected separately on each subcarrier (we term it per-subcarrier feedback). Suppose that each subcarrier is assigned B_1 feedback bits. The transceiver will need a codebook $\mathcal{T}$ of size $N = 2^{B_1}$. To minimize the system BER, the optimal precoding matrix is chosen at the receiver to minimize $\text{BER}[p]$ at the pth subcarrier as

$$\mathbf{T}^{\text{opt}}[p] = \arg\min_{\mathbf{T}[p]\in\mathcal{T}} \text{BER}[p]. \quad (9)$$

Notice that $\text{BER}[p]$ depends on the current channel realization $\mathbf{H}[p]$ and the choice of $\mathbf{T}[p]$. The index of $\mathbf{T}^{\text{opt}}[p]$ in the codebook will be fed back to the transmitter via B_1 feedback bits. With B_1 bits per subcarrier, the total needed feedback is N_cB_1 bits, which is large when N_c or B_1 is large.

III. Recursive and Trellis-Based Feedback Reduction

Feedback reduction is possible since channel responses across OFDM subcarriers are correlated; notice that frequency responses on the N_c subcarriers in (1) are decided by $L+1$ channel taps. We can view the feedback reduction problem as a *compression* type of problem, and apply tools from the source coding or vector quantization literature. We here propose two approaches, one recursive feedback encoding and the other trellis-based feedback encoding. They correspond to recursive vector quantization and trellis coded quantization, respectively [7]. For brevity, we illustrate the development for precoded spatial multiplexing.

A. Recursive Feedback Encoding

A recursive vector quantizer (VQ) is a vector quantizer with *memory*, where the quantizer output depends not only on the current input, but also on prior inputs [7]. Using state variables to summarize the influence of the past on the current operation of the quantizer, recursive VQ can be effectively described by state transition and state-dependent encoding [7]. A finite state vector quantizer (FSVQ) is simply a recursive VQ with a finite number of states.

To apply the concept of FSVQ in our problem, we need to introduce time evolution. We view the subcarrier index p as the *virtual* time index, and pursue the precoding matrices *sequentially* across the subcarriers from $p = 0$ to $p = N_c - 1$.

Denote $\xi[p]$ as the quantizer state at time p. We assume that $\xi[p]$ can take values from a finite set of states with N_{state} elements, denoted as $\{\xi_1, \ldots, \xi_{N_{\text{state}}}\}$. Given the previous state $\xi[p-1]$ and the current channel input $\mathbf{H}[p]$, we denote the state transition as

$$\xi[p] = \text{nextstate}(\xi[p-1], \mathbf{H}[p]), \quad (10)$$

where nextstate$(\cdot)$ is a function to be specified. To perform a state dependent encoding, we associate each state ξ_i with a codebook $\mathcal{T}_i$ which contains 2^{B_2} codewords, where $B_2 < B_1$. Similar to (9), the optimal precoding matrix at time p is then

$$\mathbf{T}^{\text{opt}}[p] = \arg\min_{\mathbf{T}[p]\in\mathcal{T}[p-1]} \text{BER}[p], \quad (11)$$

where $\mathcal{T}[p-1]$ stands for the current codebook associated with the state $\xi[p-1]$ known at time p, and $\text{BER}[p]$ is computed from (5) based on $\mathbf{H}[p]$ and $\mathbf{T}[p]$. Specifying $\mathbf{T}^{\text{opt}}[p]$ only requires B_2 bits, when $\mathcal{T}[p-1]$ is available.

Designing the states $\{\xi_i\}$ and the state-dependent codebooks $\{\mathcal{T}_i\}$ is an interesting problem. The optimal design may explicitly exploit the channel correlation information. We next specify one simple design based on heuristics. This design does not exploit any statistical information.

- We construct the same number of states as the codebook size of $\mathcal{T}$; hence $N_{\text{state}} = 2^{B_1}$. Each state ξ_i is characterized by one precoding matrix $\mathbf{T}_i$.
- We initialize $\xi[0]$ based on (9), that requires B_1 feedback bits.
- We construct each new codebook $\mathcal{T}_i$ as a subset of $\mathcal{T}$ as follows:

$$\mathcal{T}_i = \text{collection of } 2^{B_2} \text{ codewords from } \mathcal{T} \text{ that are closest to } \mathbf{T}_i, \quad (12)$$

  where we use the chordal distance $d_c(\mathbf{T}_i, \mathbf{T}_j) = \frac{1}{\sqrt{2}}\|\mathbf{T}_i\mathbf{T}_i^{\mathcal{H}} - \mathbf{T}_j\mathbf{T}_j^{\mathcal{H}}\|_F$ as the distance measure [6]. Chordal distance is the appropriate distance measure for column-orthonormal matrices. The codebook $\mathcal{T}$ is constructed in [10] to maximize the minimum chordal distances between any pair of codewords. Notice that $\mathcal{T}_i$ is centered around $\mathbf{T}_i$ and includes $\mathbf{T}_i$ itself, as illustrated in Fig. 2. Selecting the optimal precoding matrix as in (11) requires B_2 bits per subcarrier.
- We define the state transition as

$$\xi[p] = \xi_j, \quad \text{if } \mathbf{T}^{\text{opt}}[p] = \mathbf{T}_j. \quad (13)$$

  In such a way, the codebook $\mathcal{T}[p]$ will be centered around the most recent precoding matrix $\mathbf{T}^{\text{opt}}[p]$.

The receiver needs to feed the initial state $\xi[0]$ and the state-dependent codeword index back to the transmitter. The transmitter starts from $\xi[0]$, decides $\mathbf{T}[p]$ and $\xi[p]$ ([c.f. (13)]) based on the knowledge of $\xi[p-1]$ and the state-dependent codeword index. Following the state transition, the transmitter outputs all precoding matrices for N_c subcarriers. The total feedback required in this scheme is: $B_1 + (N_c - 1)B_2$.

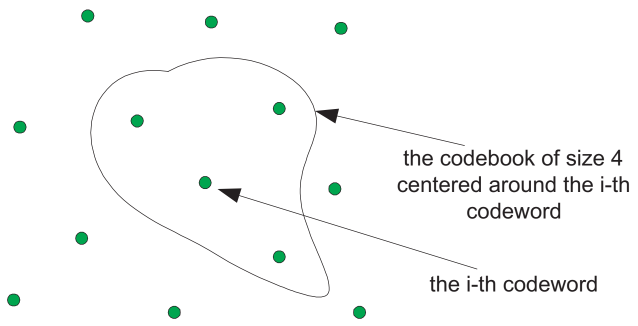

Fig. 2. Neighboring codewords form a small-size state-dependent codebook.

Our simple design here is similar to DPCM (differential pulse coded modulation). Instead of coding the codeword index itself per subcarrier, we now quantize the relative difference with respect to the previous codeword by only searching its neighbors (a total of 2^{B_2} nearest neighbors). If indeed the channel response changes slowly from subcarrier to subcarrier, the optimal codeword $\mathbf{T}[p]$ specified in (9) could be right in the neighborhood of $\mathbf{T}[p-1]$. In this case, the same performance would have been achieved with reduced feedback; however, if some abrupt change were to happen between adjacent subcarriers, the state transition might lose track, and the system performance would deteriorate considerably.

B. Trellis-Based Feedback Encoding

The drawback of the recursive feedback encoding is that the state transition may lose track from time to time. Notice that the decision on $\mathbf{T}[p]$ has only relied on prior channels inputs $\mathbf{H}[p-k]$, $k > 0$. Hence, the correlation across subcarriers is only utilized in a "causal" fashion. This causality is not necessary in MIMO-OFDM as the feedback is done on a block basis. If we follow the state transition from $p = 0$ to $p = N_c - 1$, the decision shall be made at time $p = N_c - 1$, to specify the optimal codeword indexes for all subcarriers at once. This is along the principle of tree or trellis based vector quantization [7].

We define N_{state} states as $\{\xi_i\}_{i=1}^{N_{\text{state}}}$ as before. To specify a trellis, we assume that each state is connected to 2^{B_2} next states. Hence, each state has 2^{B_2} outgoing branches, and we number them with an integer $j = 0, 1, \ldots, 2^{B_2} - 1$. Denote the state at the virtual time p as $\xi[p]$. The trellis transition corresponding to the jth branch of state $\xi[p]$ can be described by

$$\xi[p] = \text{nextstate}(\xi[p-1], j), \tag{14}$$

where $\text{nextstate}(\cdot)$ is a function to be designed. With the $\text{output}(\cdot)$ function to be specified, we denote the output of the jth branch of state $\xi[p]$ as

$$\mathbf{T}[p] = \text{output}(\xi[p-1], j). \tag{15}$$

An evolution path along the trellis will hence lead to precoding matrix for all subcarriers.

The optimal design of the trellis (14) and the output mapping (15) depend on channel characteristics. We here specify a simple design based on the nearest neighbor rule in Section III-A.

- We construct the same number of states as the codebook size of $\mathcal{T}$; hence $N_{\text{state}} = 2^{B_1}$. Each state ξ_i is characterized by one precoding matrix $\mathbf{T}_i$.
- We initialize $\xi[0]$ based on (9), that requires B_1 feedback bits.
- For each state $\xi[p]$, we define 2^{B_2} neighbor states, denoted by $\text{neighbor}(\xi_i, j)$, for $j = 0, \ldots, 2^{B_2} - 1$. For each state ξ_i, we arrange the codewords in $\mathcal{T}_i$ of (12) in descending order of the chordal distances relative to $\mathbf{T}_i$. The states corresponding to the codewords in $\mathcal{T}_i$ are the neighboring states of ξ_i. Obviously, $\text{neighbor}(\xi_i, 0) = \xi_i$, as we include ξ_i itself as its closest neighbor. The nextstate function is then simplified as

$$\xi[p] = \text{neighbor}(\xi[p-1], j), \quad j = 0, 1, \ldots, 2^{B_2} - 1. \tag{16}$$

- We define the $\text{output}(\cdot)$ in (15) as:

$$\mathbf{T}[p] = \mathbf{T}_i, \text{ if } \xi[p] = \xi_i. \tag{17}$$

- We define the branch metric from state $\xi[p-1]$ to $\xi[p]$ as

$$\text{Metric}(\xi[p-1], \xi[p]) = \frac{1}{N_c}\text{BER}\Big(\mathbf{H}[p], \text{output}(\xi[p-1], j)\Big), \tag{18}$$

where $\text{BER}(\cdot,\cdot)$ denotes the BER computed from (5) based on $\mathbf{H}[p]$ and $\mathbf{T}[p] = \text{output}(\xi[p-1], j)$.

For each path following the trellis, the resulting average BER of the system is:

$$\overline{\text{BER}} = \sum_{p=0}^{N_c-1} \text{Metric}(\xi[p-1], \xi[p]). \tag{19}$$

The best path that minimizes $\overline{\text{BER}}$ is what we are looking for. The search is easily done by the Viterbi algorithm; that is, by dynamic programming.

To recover the optimal path at the transmitter, the receiver needs to feedback the initial state $\xi[0]$ and the input branches from $p = 1$ to $p = N_c - 1$. Hence, the feedback amount is $B_1 + (N_c - 1)B_2$, the same as that of recursive encoding.

With 2^{B_1} states, 2^{B_2} branches per state, and N_c subcarriers, the trellis-based approach checks a total of $2^{B_1}(2^{B_2})^{N_c-1}$ different paths. The Viterbi complexity is at the order of $(N_c-1)2^{B_1+B_2} + 2^{B_1}$. However, we should point out that branch metrics in (18) only depend on the next state $\xi[p]$. Hence, only a total of $N_c 2^{B_1}$ different metrics are actually computed, where all incoming branches to one state share the same metric. The complexity of branch computations would be the same as the per subcarrier feedback case. The complexity increase is at the add-compare-select part of the Viterbi algorithm. Notice that the recursive coding actually follows one valid path in the trellis. Hence, the trellis-based approach outperforms recursive encoding at the expense of complexity at the receiver.

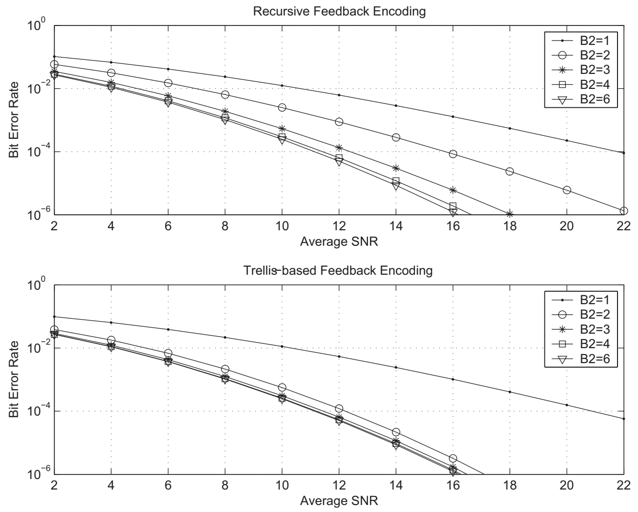

Fig. 3. Performance of the recursive and trellis based feedback reduction.

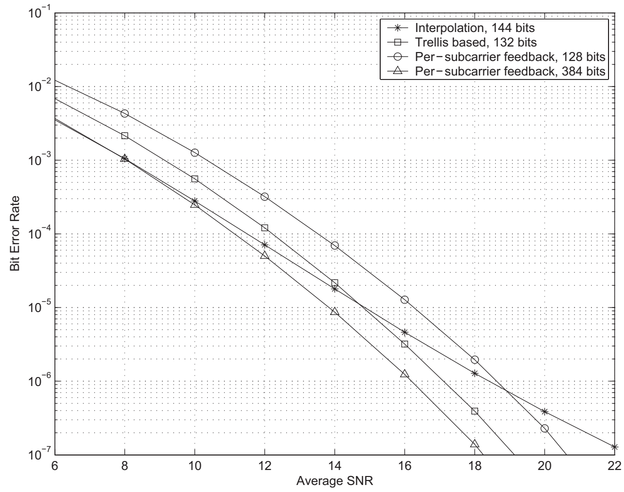

Fig. 4. The BER performance for MIMO-OFDM with transmit beamforming per subcarrier.

IV. Numerical Results

We use OFDM modulation with $N_c = 64$ subcarriers. Between each transmit-receive antenna pair, we generate the channel using the HIPERLAN/2 channel model B [5]. The channels among different antenna pairs are generated independently. Our results are based on 5000 channel realizations.

In our following plots, the average SNR is defined at each receive antenna. If the information symbols are drawn from a signal constellation with energy E_s, we have SNR $= E_s/N_0$ for the beamforming case, SNR $= N_sE_s/N_0$ for the precoded spatial multiplexing case, and SNR $= 2E_s/N_0$ for the precoded OSTBC case in (7). We use QPSK constellation throughout.

Test Case 1: Performance degradation with respect to reduced feedback. We set $N_t = 4$ and $N_r = 1$, and MIMO-OFDM uses transmit beamforming on each subcarrier. We examine the performance behavior of the proposed recursive and trellis-based methods by varying the amount of feedback. We set $B_1 = 6$, and use the beamforming codebook with size 64 from [9]. Per-subcarrier feedback will require $N_cB_1 = 384$ bits. We vary $B_2 = 1, 2, 3, 4$, so that the amount of feedback is $69, 132, 195, 258$. Fig. 3 depicts the BER performance for the recursive and trellis-based methods. When $B_1 = 1$, both methods suffer severe performance loss, as the number of neighbors is too small. As B_2 increases, the performance improves quickly. The trellis-based method achieves graceful performance degradation with feedback reduction. For example, the gap between the trellis-based method and the per subcarrier feedback is already small even with $B_2 = 2$. When $B_2 = 3$ (about 50% feedback saving), the performance degradation is negligible.

On the other hand, the recursive method works well only when the feedback reduction percentage is small. For this reason, the recursive method is not attractive for MIMO-OFDM. However, the feedback encoding for the recursive method is "causal" in the sense that later decisions only depend on current and past inputs, which might be required for some application scenarios. For example, the recursive method can be applied to a *time-selective* but frequency-flat channel, while the trellis-based method cannot.

Test Case 2: Comparison results of transmit beamforming. We set $N_t = 4$ and $N_r = 1$. Without feedback reduction, the benchmark system requires 384 bits when $B_1 = 6$. We now consider various feedback reduction alternatives: i) the trellis-based method with $B_1 = 6$ and $B_2 = 2$, that requires 132 feedback bits; ii) the interpolation-based method with 16 subcarrier groups and 3-bit phase quantization, that leads to 144 feedback bits [3]; iii) a per-subcarrier feedback with a smaller size codebook, which needs 128 feedback bits when $B_1 = 2$ (corresponding to antenna selection in this setup). The reduced feedback is about $1/3$ of the original feedback. Fig. 4 depict the performance for those competing schemes. We observe that:

- O1: The interpolation-based method has excellent performance at low SNR. However, as the SNR increases, the BER curve of the interpolation-based method levels off, indicating "diversity loss". This observation has already been made in [3]. Diversity loss leads to severe performance degradation at high SNR.
- O2: The trellis-based method differs from the benchmark performance by a constant amount (about 0.9 dB) throughout the SNR range. It is slightly inferior to the interpolation-based method at low SNR, but outperforms the latter considerably at high SNR, since it does not suffer from "diversity loss". The trellis-based method outperforms per-subcarrier feedback by a constant amount (about 1.5 dB).

Test Case 3: Comparison results of precoded spatial multiplexing. We set $N_t = 4$, $N_r = 2$, and $N_s = 2$. We compare similar setups as in the beamforming case except that the interpolation method now uses 2-bit quantization on the rotation matrix in [4] (thus 128 feedback bits). The precoder codebooks with size 64 and 4 are taken from [10]. From Fig. 5, we have observations similar to those in O1-O2. The relative positions of the curves slightly change. The gap between the trellis-based method and the per-subcarrier feedback with a smaller-size codebook increases to about 2dB. The advantage of the trellis-based method relative to the interpolation-based approach is more evident in this setting.

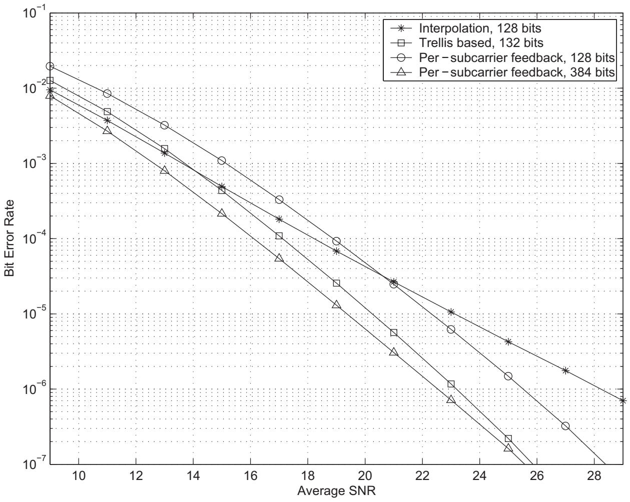

Fig. 5. The BER performance for MIMO-OFDM with precoded spatial multiplexing per subcarrier

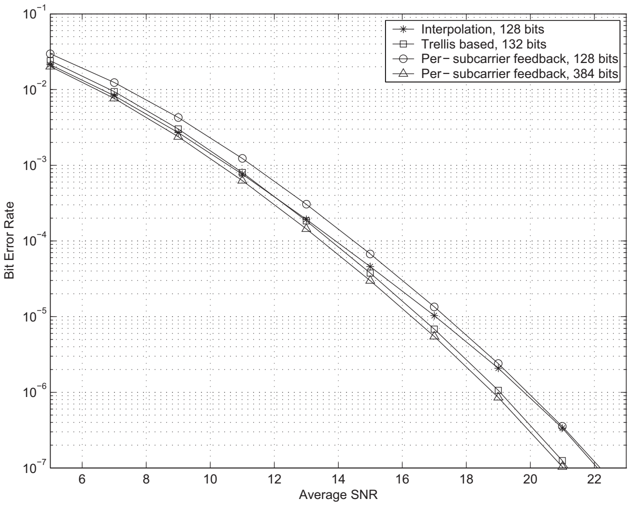

Fig. 6. The BER performance for MIMO-OFDM with precoded OSTBC per subcarrier

Test Case 4: Comparison results of precoded OSTBC. We set $N_t = 4$ and $N_r = 1$. The precoded Alamouti STBC is applied on each OFDM subcarrier. The codebooks are the same as in Test Case 2. Based on Fig. 6, we have observations similar to those in O1-O2. The gaps between the different curves decrease considerably, revealing that orthogonal STBC is less sensitive to feedback imperfection.

V. CONCLUSIONS

In this paper, we considered adaptive MIMO-OFDM with three different transmitter configurations: i) beamforming applied per subcarrier, ii) precoded spatial multiplexing applied per subcarrier, and iii) precoded orthogonal STBC applied per subcarrier. We proposed two methods to reduce the amount of feedback, one recursive feedback encoding, and the other trellis-based feedback encoding. The trellis based approach achieves considerable feedback reduction with graceful performance degradation. Our numerical results demonstrate that the trellis-cased method is inferior to an existing interpolation-based alternative only slightly at low SNR, but outperforms the latter considerably at high SNR, as it does not suffer from "diversity loss".

Our designs relied on tools from the vector quantization literature, by viewing our problem at hand as a compression type of problem. Our recursive and trellis-based designs used only simple nearest neighbor rules; utilization of statistical channel information may be beneficial. Finally, trellis design based on set partitioning may further improve performance, as with trellis coded modulation or trellis coded quantization [7].

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