2005 IEEE 6th Workshop on Signal Processing Advances in Wireless Communications

BER CRITERION AND CODEBOOK CONSTRUCTION FOR FINITE-RATE PRECODED SPATIAL MULTIPLEXING

Shengli Zhou and Baosheng Li

Department of Electrical and Computer Engineering, University of Connecticut, Storrs, Connecticut 06269, USA

ABSTRACT

Precoded spatial multiplexing systems with rate-limited feedback have been studied recently based on various precoder selection criteria. Instead of those based on indirect performance indicators, we in this paper propose a new criterion directly based on the exact bit error rate (BER) of the system. We show that in the asymptotic case of infinite-rate feedback, the optimal precoder design based on the BER criterion is drastically different from the counterparts with other criteria. In the finite-rate case, we apply the generalized Lloyd algorithm used in vector quantization to construct good precoder codebooks. The new found codebooks improve considerably the minimum distance relative to those currently available. Our numerical results compare the BER performance with different codebooks and with different selection criteria.

1. INTRODUCTION

Multi-antenna diversity has by now well established as an effective fading counter-measure for wireless communications. To further improve system performance, the receiver can feedback channel state information (CSI) back to the transmitter, so that the transmission parameters such as power and modulation type can be adapted to the channel. In practical wireless systems, however, the CSI at the transmitter suffers from imperfections originating from various sources, such as estimation errors, feedback delay and feedback errors. These considerations have sparked recent research interests towards quantifying and exploiting imperfect (or partial) CSI in multi-antenna systems.

Partial CSI can be modeled in different ways [12]. One class of CSI models impose a bandwidth constraint on the feedback channel which is only able to communicate a finite number of bits per block. Finite-rate transmit beamforming has been investigated based on various criteria such as the average signal to noise ratio (SNR) [12, 10], the outage probability [11], and the symbol error rate [19], respectively. Subject to finite-rate feedback, optimal transmission is also pursued in [5, 2, 13] to maximize the average channel capacity, while adaptive modulation together with transmit beamforming has been pursued in [16] to enhance the transmission rate. Recently, the application of finite rate feedback in a precoded spatial multiplexing system has been addressed in [8, 7], where various criteria on precoder selection and codebook construction have been proposed.

As in [8, 7], we here investigate precoded spatial multiplexing with finite-rate feedback. Our contributions are as follows.

- Instead of those criteria based on indirect performance indicators [8, 7], we propose a new precoder selection criterion directly based on the exact BER of the system. The new criterion hence outperforms competing alternatives in terms of improving the system BER performance.
- In the asymptotic case with infinite-rate feedback, we characterize the optimal precoder based on the BER criterion. Our precoder design is drastically different from the counterparts based on all other criteria [8, 7].
- In the finite-rate case, we apply the generalized Lloyd algorithm used in vector quantization [4] to construct good precoder codebooks. The new found codebooks improve considerably the distance property relative to those currently available.

Notation: Bold upper and lower letters denote matrices and column vectors, respectively; $(\cdot)^T$, $(\cdot)^*$, and $(\cdot)^{\mathcal{H}}$ denote transpose, conjugate, and Hermitian transpose, respectively; $|\cdot|$ stands for the absolute value of a scalar; $\mathrm{tr}(\cdot)$ and $\det(\cdot)$ stand for the trace and the determinant of a matrix, respectively. $\|\cdot\|_2$ denotes the two-norm of a vector or a matrix, while $\|\cdot\|_F$ is the Frobenius norm of a matrix. $\mathbf{I}_N$ is the $N \times N$ identity matrix; $[\mathbf{A}]_{i,j}$ stands for the $(i,j)^{\text{th}}$ entry of a matrix $\mathbf{A}$.

2. SYSTEM MODEL AND BER-BASED SELECTION CRITERION

As depicted in Fig. 1, we consider a precoded spatial multiplexing system where the transmitter and the receiver have N_t and N_r antennas, respectively. The information symbol block $\mathbf{s} := [s_1, \ldots, s_K]^T$ is precoded by a $N_t \times K$ matrix $\mathbf{T}$ to obtain the precoded block $\mathbf{Ts}$, whose N_t entries are then transmitted through N_t antennas simultaneously. Denote $h_{\nu\mu}$ as the channel coefficient between the νth receive- and the μth transmit- antenna, and collect the N_rN_t channel coefficients into the $N_r \times N_t$ channel matrix $\mathbf{H}$ with $[\mathbf{H}]_{\nu,\mu} = h_{\nu\mu}$. The received samples on N_r receive-antennas, collected in the vector $\mathbf{y}$, can be expressed as:

$$\mathbf{y} = \mathbf{HTs} + \mathbf{v}, \tag{1}$$

where $\mathbf{v}$ is the additive white Gaussian noise (AWGN) with each entry having variance N_0.

As in [7, 8], we assume that the receiver is able to feedback a finite number of (say B) bits back to the transmitter, and that the feedback link is error-free and delay-free. Under the constraint of B feedback bits, the system only needs to prepare a total of $N = 2^B$ precoding matrices. Let us denote these matrices as $\mathbf{T}_1, \mathbf{T}_2, \ldots, \mathbf{T}_N$, and collect them into a codebook $\mathcal{T}$ as:

$$\mathcal{T} := \{\mathbf{T}_1, \ldots, \mathbf{T}_N\}. \tag{2}$$

Based on the current channel realization, the receiver will decide which codeword (precoder) from the codebook $\mathcal{T}$ is the most favorable, and inform the transmitter to switch to that precoder by

* Supported by UConn Research Foundation internal grant 445157.

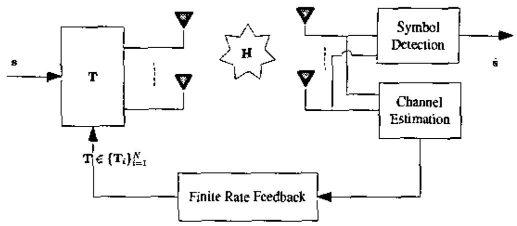

Fig. 1. Precoded spatial multiplexing with finite-rate feedback

feeding back its B-bits codeword index. For such a precoded spatial multiplexing system with finite-rate feedback, the following two important questions need to be addressed: i) how does the receiver select a favorable precoder from the codebook $\mathcal{T}$? ii) how does the system construct a good codebook $\mathcal{T}$?

These two design issues have been well addressed in [7, 8]. We next briefly summarize their results.

2.1. Brief Summary of Existing Results

First, the precoding matrices have been constrained to have orthonormal columns: $\mathbf{T}_i^{\mathcal{H}}\mathbf{T}_i = \mathbf{I}_K, i = 1, \ldots, N$ [7, 8]. With such precoders, various precoder selection criteria have been proposed in [7, 8].

- For a maximum likelihood (ML) receiver, the precoder is chosen to either maximize the minimum receiver symbol vector distance (MD Selection), or, maximize the instantaneous capacity (Capacity Selection).
- For a linear zero-forcing (ZF) receiver, the precoder is chosen to maximize the minimum singular value of $\mathbf{HT}$ (SV Selection).
- For a linear minimum mean square error (MMSE) receiver, the precoder is chosen to either minimize the trace of the mean square error matrix (MMSE-trace Selection), or, minimize the determinant of the mean square error matrix (MMSE-det Selection).

On the codebook design, the following results are available [7, 8].

- In the asymptotic case with infinite-rate feedback where $B = \infty$, the optimal precoder consists of the K eigenvectors of $\mathbf{H}^{\mathcal{H}}\mathbf{H}$ corresponding to the K largest eigenvalue, for all selection criteria in [7, 8]. To be more specific, denote the eigen decomposition of $\mathbf{H}^{\mathcal{H}}\mathbf{H}$ as:

$$\mathbf{H}^{\mathcal{H}}\mathbf{H} = \mathbf{V}_H \mathbf{\Lambda}_H \mathbf{V}_H^{\mathcal{H}}, \tag{3}$$

  where $\mathbf{\Lambda}_H = \text{diag}(\lambda_1, \ldots, \lambda_{N_t})$ contains on its diagonal the eigenvalues arranged in a non-increasing order: $\lambda_1 \geq \cdots \geq \lambda_{N_t}$. The optimal precoder is then

$$\mathbf{T}_{\text{opt}} = \overline{\mathbf{V}}_H, \tag{4}$$

  where $\overline{\mathbf{V}}_H$ consists of the first K columns of $\mathbf{V}_H$.

- In the finite-rate feedback case, if MMSE-det, or, Capacity Selection is used, the codebook $\mathcal{T}$ shall be designed to maximize $\min_{1\leq i<j\leq N} d_{\text{FS}}(\mathbf{T}_i, \mathbf{T}_j)$, where $d_{\text{FS}}(\mathbf{T}_i, \mathbf{T}_j)$ is the Fubini-Study distance defined for two subspaces spanned by $\mathbf{T}_i$ and $\mathbf{T}_j$ [1]:

$$d_{\text{FS}}(\mathbf{T}_i, \mathbf{T}_j) = \arccos|\det(\mathbf{T}_i^{\mathcal{H}}\mathbf{T}_j)|. \tag{5}$$

- In the finite-rate feedback case, if MMSE-trace, SV, or, MD Selection is used, the codebook $\mathcal{T}$ shall be designed to maximize $\min_{1\leq i<j\leq N} d_{\text{p2}}(\mathbf{T}_i, \mathbf{T}_j)$, where $d_{\text{p2}}(\mathbf{T}_i, \mathbf{T}_j)$ is the projection two-norm subspace distance defined as [1]:

$$d_{\text{p2}}(\mathbf{T}_i, \mathbf{T}_j) := \|\mathbf{T}_i\mathbf{T}_i^{\mathcal{H}} - \mathbf{T}_j\mathbf{T}_j^{\mathcal{H}}\|_2. \tag{6}$$

In a different scenario, finite rate precoding has been applied in an orthogonal space time block coded system [9], where the codebook was proposed to maximize $\min_{1\leq i<j\leq N} d_c(\mathbf{T}_i, \mathbf{T}_j)$, where $d_c(\mathbf{T}_i, \mathbf{T}_j)$ is the chordal subspace distance defined as [1]:

$$d_c(\mathbf{T}_i, \mathbf{T}_j) = \frac{1}{\sqrt{2}}\|\mathbf{T}_i\mathbf{T}_i^{\mathcal{H}} - \mathbf{T}_j\mathbf{T}_j^{\mathcal{H}}\|_F. \tag{7}$$

Sample codebooks are provided in [6].

2.2. BER-based Selection Criterion for Linear Receivers

Different from all selection criteria in [7, 8], we propose to directly use the exact BER as the selection criterion. Let us denote $\overline{\text{BER}}(\mathbf{H}, \mathbf{T})$ as the BER averaged over K data streams when the channel realization is $\mathbf{H}$ and the precoder is $\mathbf{T}$. The proposed BER-based selection rule is then

$$\mathbf{T}_{\text{opt}} = \arg\min_{\mathbf{T}\in\mathcal{T}} \overline{\text{BER}}(\mathbf{H}, \mathbf{T}), \tag{8}$$

whose codeword index is feedback to the transmitter.

The BER expression for an ML receiver is not available up to date. Hence, the BER based criterion is not applicable to an ML receiver. However, the BER for linear receivers can be easily computed thanks to the recent results in [3, 14]. We thus focus our attention on linear receivers.

2.2.1. Linear ZF receiver

The linear ZF receiver is $\mathbf{G}^{\text{zf}} = [\mathbf{T}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{HT}]^{-1}\mathbf{T}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}$. Applying the ZF receiver on (1), we obtain:

$$\mathbf{z} = \mathbf{G}^{\text{zf}}\mathbf{y} = \mathbf{s} + \mathbf{G}^{\text{zf}}\mathbf{n}, \tag{9}$$

where the processed noise has variance $N_0\mathbf{G}^{\text{zf}}(\mathbf{G}^{\text{zf}})^{\mathcal{H}} = N_0[\mathbf{T}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{HT}]^{-1}$. With E_s denoting the average symbol energy for each symbol s_k, the SNR for the kth data stream is:

$$\gamma_k^{\text{zf}} = \frac{E_s}{N_0[\mathbf{T}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{HT}]_{k,k}^{-1}} = \frac{\zeta}{[\mathbf{T}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{HT}]_{k,k}^{-1}}, \tag{10}$$

where for notational brevity we define

$$\zeta = \frac{E_s}{N_0}. \tag{11}$$

Let BER(γ) denote the relationship between BER and SNR γ in an AWGN channel. We consider the square quadrature-amplitude-modulation (QAM) with size M. The closed-form expression for BER(γ) is [3]:

$$\text{BER}(\gamma) = \frac{1}{\log_2\sqrt{M}} \sum_{k=1}^{\log_2\sqrt{M}} \frac{1}{\sqrt{M}} \sum_{i=0}^{(1-2^{-k})\sqrt{M}-1} \times \left\{(-1)^{\lfloor\frac{i\cdot 2^{k-1}}{\sqrt{M}}\rfloor}\left(2^{k-1} - \left\lfloor\frac{i\cdot 2^{k-1}}{\sqrt{M}} + \frac{1}{2}\right\rfloor\right) \cdot 2Q\left((2i+1)\sqrt{\frac{3}{M-1}\gamma}\right)\right\}, \tag{12}$$

where $Q(\cdot)$ denotes the Gaussian-Q function. For pulse-amplitude-modulation (PAM) and rectangular QAMs, closed-form BER expressions are also available [3]. For 4-QAM, eq. (12) is simply:

$$\text{BER}(\gamma) = Q(\sqrt{\gamma}). \tag{13}$$

Alternatively, one can compute $\text{BER}(\gamma)$ easily using a simple recursive algorithm in [14]. The average BER over K data streams is then

$$\overline{\text{BER}}^{\text{zf}}(\mathbf{H}, \mathbf{T}) = \frac{1}{K}\sum_{k=1}^{K} \text{BER}(\gamma_k^{\text{zf}}). \tag{14}$$

2.2.2. Linear MMSE receiver

Now let us consider the linear MMSE receiver

$$\mathbf{G}^{\text{mmse}} = \left[\mathbf{T}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{H}\mathbf{T} + (1/\zeta)\mathbf{I}_K\right]^{-1}\mathbf{T}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}. \tag{15}$$

The signal to interference-plus-noise ratio (SINR) after MMSE equalization is

$$\gamma_k^{\text{mmse}} = \frac{\zeta}{[\mathbf{T}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{H}\mathbf{T} + (1/\zeta)\mathbf{I}_K]^{-1}_{k,k}} - 1 \tag{16}$$

The residual interference-plus-noise can be well approximated by a Gaussian random variable, that leads to an average BER as

$$\overline{\text{BER}}^{\text{mmse}}(\mathbf{H}, \mathbf{T}) = \frac{1}{K}\sum_{k=1}^{K} \text{BER}(\gamma_k^{\text{mmse}}). \tag{17}$$

With any given codebook $\mathcal{T}$, the BER-based selection criterion shall outperform all other criteria in [7, 8], in terms of the uncoded BER performance, when linear receivers are used. More interestingly, we show next that the proposed BER-based criterion entails a drastically different optimal codebook in the asymptotic case with infinite-rate feedback, from those in [8].

3. THE INFINITE-RATE CODEBOOK CONSTRUCTION

We first consider the codebook construction with $B = \infty$. In this case, we assume that the transmitter has full knowledge of the channel $\mathbf{H}$, and select $\mathbf{T}$ directly based on $\mathbf{H}$. As the BER in (12) depends on the constellation, we specify our results for three different constellations: 4-QAM, 16-QAM, and 64-QAM; see [18] for results on rectangular QAMs.

Denote $\mathbf{\Theta}_{\text{cm}}$ as a $K \times K$ unitary matrix with each entry having constant modulus $1/\sqrt{K}$; e.g., a normalized FFT or Hadamard matrix. We have the following results [18].

Result 1 *With linear ZF receivers and using the BER-based selection criterion, the optimal column-orthonormal precoder* $\mathbf{T}$ *depends on the channel* $\mathbf{H}$ *as follows*

$$\mathbf{T}_{\text{opt}} = \begin{cases} \overline{\mathbf{V}}_H & \text{when } \lambda_1\zeta \le \frac{1}{x_{\text{th}}} \\ \overline{\mathbf{V}}_H\mathbf{\Theta}_{\text{cm}} & \text{when } \lambda_K\zeta \ge \frac{1}{x_{\text{th}}} \\ \text{unclear} & \text{other cases} \end{cases}, \tag{18}$$

where x_{th} *is a constellation specific constant as*

$$x_{\text{th}} = \begin{cases} 1/3 & \text{for 4-QAM} \\ 0.0667 & \text{for 16-QAM} \\ 0.0159 & \text{for 64-QAM} \end{cases}. \tag{19}$$

The quantities λ_k, $\overline{\mathbf{V}}_H$, ζ *are defined in* (3), (4),(11), *respectively.*

Result 2 *With linear MMSE receivers and using the BER-based selection criterion, the optimal column-orthonormal precoder* $\mathbf{T}$ *depends on the channel* $\mathbf{H}$ *as follows.*

1. *For 4-QAM, the optimal precoder is always*

$$\mathbf{T}_{\text{opt}} = \overline{\mathbf{V}}_H\mathbf{\Theta}_{\text{cm}}. \tag{20}$$

2. *For 16-QAM and 64-QAM, the optimal precoder is*

$$\mathbf{T}_{\text{opt}} = \begin{cases} \overline{\mathbf{V}}_H & \text{when } (\frac{1}{x_{\text{th},2}} - 1) \le \lambda_K\zeta \\ & \text{and } \lambda_1\zeta \le (\frac{1}{x_{\text{th},1}} - 1) \\ \overline{\mathbf{V}}_H\mathbf{\Theta}_{\text{cm}} & \text{when } \lambda_K\zeta \ge (\frac{1}{x_{\text{th},1}} - 1) \\ & \text{or, } \lambda_1\zeta \le (\frac{1}{x_{\text{th},2}} - 1) \\ \text{unclear} & \text{other cases} \end{cases}, \tag{21}$$

where $x_{\text{th},1}$ *and* $x_{\text{th},2}$ *are two constants as:*

$$(x_{\text{th},1}, x_{\text{th},2}) = \begin{cases} (0.0683, 0.8025) & \text{for 16-QAM} \\ (0.0159, 0.7907) & \text{for 64-QAM} \end{cases} \tag{22}$$

The quantities λ_k, $\overline{\mathbf{V}}_H$, ζ *are defined in* (3), (4),(11), *respectively.*

When $\mathbf{T} = \overline{\mathbf{V}}_H$, we have $\gamma_k^{\text{zf}} = \gamma_k^{\text{mmse}} = \zeta\lambda_k, \forall k = 1, \ldots, K$. When $\mathbf{T} = \overline{\mathbf{V}}_H\mathbf{\Theta}_{\text{cm}}$, we have

$$\gamma_1^{\text{zf}} = \cdots = \gamma_K^{\text{zf}} = \frac{K\zeta}{\frac{1}{\lambda_1} + \ldots + \frac{1}{\lambda_K}}, \tag{23}$$

$$\gamma_1^{\text{mmse}} = \cdots = \gamma_K^{\text{mmse}} = \frac{K}{\frac{1}{1+\zeta\lambda_1} + \ldots + \frac{1}{1+\zeta\lambda_K}} - 1. \tag{24}$$

As we can see, the behavior for the optimal codebook is drastically different from those in [7, 8].

Since optimal precoder is still unclear for certain cases, we propose the following practical solution, which avoids the need to compute the constellation specific thresholds.

Proposition 1 *With linear ZF or MMSE receivers, for each channel realization* $\mathbf{H}$, *the transmitter selects the precoder to be either* $\overline{\mathbf{V}}_H$, *or,* $\overline{\mathbf{V}}_H\mathbf{\Theta}_{\text{cm}}$, *depending on which one yields better BER performance.*

We next offer new codebook construction when B is finite.

4. THE FINITE-RATE CODEBOOK CONSTRUCTION

Using the generalized Lloyd algorithm to search for finite-rate beamforming codebook was first used in [12] with $N_t = 2$. It is later used in [15] to design finite-rate beamforming codebook for both i.i.d. fading channels and correlated fading channels, with an arbitrary N_t. We in this paper apply the Lloyd algorithm to search for good precoder codebooks.

The codebook design of $\mathcal{T}$ is linked to a vector quantization problem as follows. Suppose we have a random $N_t \times K$ matrix $\mathbf{V}$, which is isotropically distributed. We now want to quantize $\mathbf{V}$ to a finite number of codewords that form $\mathcal{T}$. We adopt the chordal distance as the distance metric. Our objective is to minimize the average distortion defined as:

$$J = E\{\min_{1\le i\le N} d_c^2(\mathbf{V}, \mathbf{T}_i)\} = \sum_{i=1}^{N} \text{E}_{\mathcal{R}_i}\{d_c^2(\mathbf{V}, \mathbf{T}_i)\}P(\mathbf{V} \in \mathcal{R}_i), \tag{25}$$

where $E\{\cdot\}$ stands for expectation, $P(\mathbf{V} \in \mathcal{R}_i)$ is the probability of a random $\mathbf{V}$ belonging to the region $\mathcal{R}_i$, which is defined as

$$\mathcal{R}_i = \{\mathbf{V} | d_c(\mathbf{V}, \mathbf{T}_i) < d_c(\mathbf{V}, \mathbf{T}_j), \quad \forall j \neq i\}. \tag{26}$$

The Lloyd algorithm can iteratively reduce the cost function in (25). The reason that we choose the chordal distance is that it can be re-expressed as $d_c^2(\mathbf{T}_i, \mathbf{T}_j) = \text{tr}(\mathbf{I}_K - \mathbf{T}_j^{\mathcal{H}}\mathbf{T}_i\mathbf{T}_i^{\mathcal{H}}\mathbf{T}_j)$, that will render simple analytical solution inside the iterations of the Lloyd algorithm.

The codebook design steps are as follows.

S1) To avoid the expectation operation in (25), we use the Monte-Carlo approach, as in [15]. We generate a training set with N_{tr} samples $\{\mathbf{V}_n\}_{n=1}^{N_{tr}}$.

S2) Starting with an initial codebook (obtained via random computer search, or, using the currently best codebook if available), we carry out the following two sub-steps iteratively.

- Nearest neighbor rule [4]: assign $\mathbf{V}_n$ to one of the regions using the rule

$$\mathbf{V}_n \in \mathcal{R}_i \quad \text{if} \quad d_c(\mathbf{V}_n, \mathbf{T}_i) < d_c(\mathbf{V}_n, \mathbf{T}_j), \forall j \neq i$$

- Centroid condition [4]: For each region $\mathcal{R}_i$, find the optimal codebook as

$$\begin{aligned} \mathbf{T}_i^{\text{opt}} &= \arg\min_{\mathbf{T}} \frac{1}{N_{tr}} \sum_{\mathbf{V}_n \in \mathcal{R}_i} d_c^2(\mathbf{V}_n, \mathbf{T}) \\ &= \arg\min_{\mathbf{T}} \frac{1}{N_{tr}} \sum_{\mathbf{V}_n \in \mathcal{R}_i} \text{tr}(\mathbf{I}_K - \mathbf{T}^{\mathcal{H}}\mathbf{V}\mathbf{V}^{\mathcal{H}}\mathbf{T}) \\ &= \arg\max_{\mathbf{T}} \text{tr}(\mathbf{T}^{\mathcal{H}}\mathbf{R}\mathbf{T}), \end{aligned} \tag{27}$$

  where $\mathbf{R}$ is defined as $\mathbf{R} = \frac{1}{N_{tr}} \sum_{\mathbf{V}_n \in \mathcal{R}_i} \mathbf{V}_n\mathbf{V}_n^{\mathcal{H}}$. It is easy to show that $\mathbf{T}_i^{\text{opt}}$ shall be taken as the K eigenvectors of $\mathbf{R}$ corresponding to the K largest eigen values.

Notice that the Lloyd algorithm converges with J monotonically decreasing. But this does not mean that the minimum distance of the codebook is monotonically improving, as observed in [15]. During each iteration, we examine the tentative codebook, and record it if its minimum distance is larger than the currently best. This is done for each distance d_c, d_{FS}, and d_{p2}.

S3) Go back to S1 to generate another training set, and re-run the Lloyd algorithm in S2. We stop until no further improvement on the minimum distance is observed.

5. NUMERICAL RESULTS

We focus on the configuration with $(N_t, N_r, K) = (6, 3, 3)$, and adopt 4-QAM constellation. In all plots, we define the average SNR at each receive antenna as: SNR $= \frac{KE_s}{N_0} = K\zeta$.

Test Case 1 (infinite-rate feedback): We first test our theoretical analysis in Section 3 where $B = \infty$. We observe from Fig. 2 that the ZF receiver with the precoder $\mathbf{T} = \overline{\mathbf{V}}_H$ outperforms that with $\mathbf{T} = \overline{\mathbf{V}}_H\Theta_{\text{cm}}$ at low SNR, but vice versa as high SNR. The proposed solution in Proposition 1 outperforms either of them by always choosing the better one for every channel realization. With 4-QAM, we observe from Fig. 2 that the MMSE receiver with $\mathbf{T} = \overline{\mathbf{V}}_H\Theta_{\text{cm}}$ always outperforms that with $\mathbf{T} = \overline{\mathbf{V}}_H$, agreeing with our theoretical analysis. See [18] for results on 16-QAM.

Test Case 2 (codebook construction by the Lloyd algorithm): We collect the codebooks obtained via the Lloyd algorithm for $(N_t, K) = (6, 3)$ in the following, where we use boldface fonts to highlight the maximized minimum distances d_c, d_{FS}, or d_{p2}, if multiple codebooks are listed for one configuration.

	Codebooks listed in [6] $(d_c, d_{\text{FS}}, d_{\text{p2}})$	New codebooks $(d_c, d_{\text{FS}}, d_{\text{p2}})$
B=4	(1.0922, 1.1936, 0.8115)	(**1.2281**, 1.3259, 0.8881)
		(1.1740, **1.3548**, 0.9100)
		(1.1782, 1.3218, **0.9314**)
B=5	(1.0156, 1.0724, 0.7472)	(**1.1539**, 1.2500, 0.8636)
		(1.1424, **1.2730**, 0.8748)
		(1.1401, 1.2635, **0.8830**)
B=6	(**0.9324**, 0.9722, 0.6827)	(**1.0625**, 1.1004, 0.7637)
	(0.8959, **0.9869**, 0.7097)	(1.0216, **1.1710**, 0.7813)
	(0.8834, 0.9559, **0.7508**)	(1.0242, 1.1529, **0.8239**)

The new codebooks (list in [17]) have much larger minimum distances than the codebooks currently available in [6]. This demonstrates the effectiveness of the Lloyd algorithm for finding good codebooks.

Test Case 3 (Comparison among different selection criteria): We now compare the proposed BER criteria with those in [8]. We deploy codebooks optimized based on the chordal distance. Fig. 3 compares the SV and the BER selection criteria for ZF receivers. We observe that the SV criterion yields performance quite close to the BER criteria. This is reasonable, as the SV criterion tries to improve the worst SNR for K data streams, implicitly enforcing some averaging over all subchannel SNRs. On the other hand, Fig. 4 compares the MMSE-trace, MMSE-det, the BER criteria for MMSE receivers. We observe that MMSE-det performs worse than the MMSE-trace criterion, and both of them are inferior to the BER criterion. Therefore, the BER based selection criterion has the performance advantage over competing alternatives.

Test Case 4: We now test the performance improvement as a function of the number of feedback bits, as reported in Fig. 5 for ZF receivers. The $B = 0$ case corresponds to a $N_r \times N_r$ MIMO system, where linear receivers can be applied. The $B = 1$ case corresponds to antenna subset selection, with N_t antennas partitioned to two sets, each with K elements. The asymptotic case with $B = \infty$ is also included as a performance benchmark. We observe: i) feedback link improves the system performance drastically; ii) the performance gain demonstrates diminishing returns as the number of feedback bits increases; iii) The large portion of the feedback gain is achieved with only moderate number of bits, (e.g., $B = 6$). Hence, *the number of feedback bits in practical systems needs not be large.*

6. ACKNOWLEDGEMENT

The authors are grateful to Mr. Pengfei Xia for helpful discussions.

7. REFERENCES

[1] A. Barg and D. Yu. Nogin, "Bounds on packings of spheres in the Grassmann manifold," *IEEE Trans. on IT*, Sept. 2002.

[2] R. S. Blum, "MIMO with limited feedback of channel state information," in *Proc. of ICASSP*, Apr. 2003, pp. 89–92.

[3] K. Cho and D. Yoon, "On the general BER expression of one- and two-dimensional amplitude modulations," *IEEE Trans. on Commu.*, vol. 50, pp. 1074–1080, July 2002.

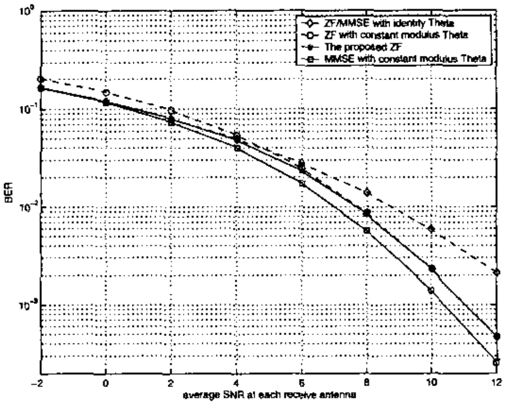

Fig. 2. Performance comparison with infinite-rate feedback

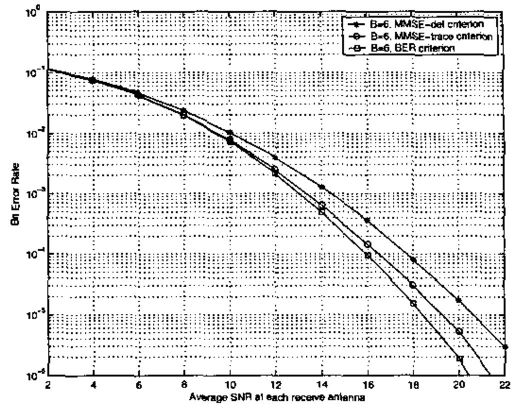

Fig. 4. MMSE receivers with different precoder selection criteria

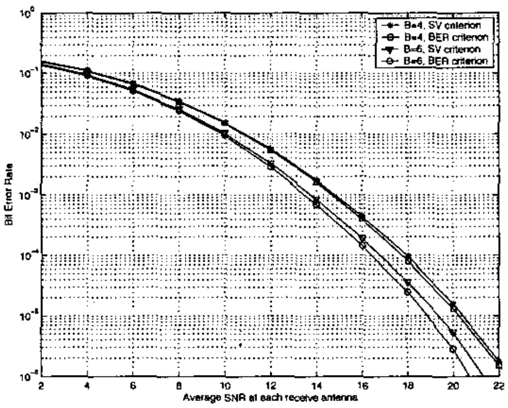

Fig. 3. ZF receiver: SV versus BER criteria

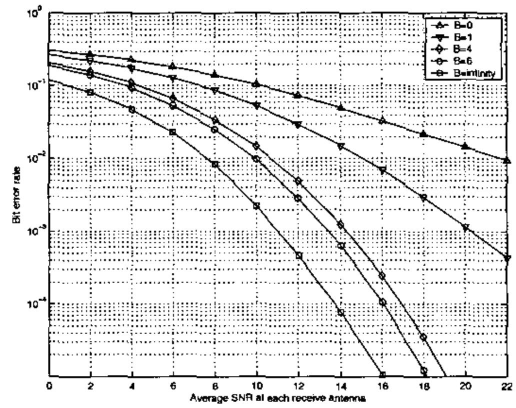

Fig. 5. Performance improvement with # of feedback bits, ZF

[4] A. Gersho and R. M. Gray, *Vector Quantization and Signal Compression*, Kluwer Academic Publishers, 1992.

[5] V. Lau, Y. Liu, and T.-A. Chen, "On the design of MIMO block-fading channels with feedback-link capacity constraint," *IEEE Trans. on Commun.*, vol. 52, Jan. 2004.

[6] D. Love, *Personal Webpage on Grassmannian Subspace Packing*, http://dynamo.ecn.purdue.edu/~djlove/grass.html.

[7] D. J. Love and R. W. Heath Jr., "Grassmannian precoding for spatial multiplexing systems," in *Proc. of Allerton Conf. on Comm. Control and Comp.*, Monticello, IL, Oct. 1–3 2003.

[8] D. J. Love and R. W. Heath Jr., "Limited feedback precoding for spatial multiplexing systems," in *Proc. of Globecom*, Dec. 2003, vol. 4, pp. 1857–1861.

[9] D. J. Love and R. W. Heath Jr., "Limited Feedback Precoding for Orthogonal Space-Time Block Codes," in *Proc. of Globecom*, Dallas, TX, Nov. 29 - Dec. 3 2004.

[10] D. J. Love, R. W. Heath Jr., and T. Strohmer, "Grassmannian beamforming for multiple-input multiple-output wireless systems," *IEEE Trans. on IT*, Oct. 2003.

[11] K. K. Mukkavilli, A. Sabharwal, E. Erkip, and B. Aazhang, "On Beamforming with Finite Rate Feedback in Multiple Antenna Systems," *IEEE Trans. on IT*, Oct. 2003.

[12] A. Narula, M. J. Lopez, M. D. Trott, and G. W. Wornell, "Efficient use of side information in multiple-antenna data transmission over fading channels," *IEEE JSAC*, Oct. 1998.

[13] W. Santipach and M. L. Honig, "Asymptotic performance of MIMO wireless channels with limited feedback," in *Proc. of MILCOM Conf.*, Oct. 13-16 2003, vol. 1, pp. 141–146.

[14] P. K. Vitthaladevuni and M.-S. Alouini, "A recursive algorithm for the exact BER computation of generalized hierarchical QAM constellations," *IEEE Trans. on IT*, Jan. 2003.

[15] P. Xia and G. B. Giannakis, "Design and analysis of transmit-beamforming based on limited-rate feedback," in *Proc. of the Vehicular Technology Conf.*, Los Angeles, CA, Sept. 2004.

[16] P. Xia, S. Zhou, and G. B. Giannakis, "Multi-Antenna Adaptive Modulation with Beamforming based on Bandwidth-Constrained Feedback," *IEEE Transactions on Communications*, vol. 53, no. 3, pp. 526-536, March 2005.

[17] S. Zhou, *Personal Webpage on Collected Codebooks*, http://engr.uconn.edu/~shengli/codebooks.

[18] S. Zhou, and B. Li, "BER Criterion and Codebook Construction for Finite-Rate Precoded Spatial Multiplexing with Linear Receivers", *IEEE Trans. on SP*, revised March 2005.

[19] S. Zhou, Z. Wang, and G. B. Giannakis, "Quantifying the power-loss when transmit-beamforming relies on finite rate feedback," *IEEE Trans. Wireless Commun.*, 2004 (to appear).